THE BIG IDEA

An object will remain in rotational equilibrium if its center of mass is above the area of support.

What determines whether an object will rotate when a force acts on it?

Why doesn't the Leaning Tower of Pisa rotate and topple over?

What maneuvers does a falling cat make to land on its feet?

This chapter is about the factors that affect rotational equilibrium.

11.1 Torque

To make an object turn or rotate, apply a torque.

11.1 Torque

Every time you open a door, turn on a water faucet, or tighten a nut with a wrench, you exert a turning force.

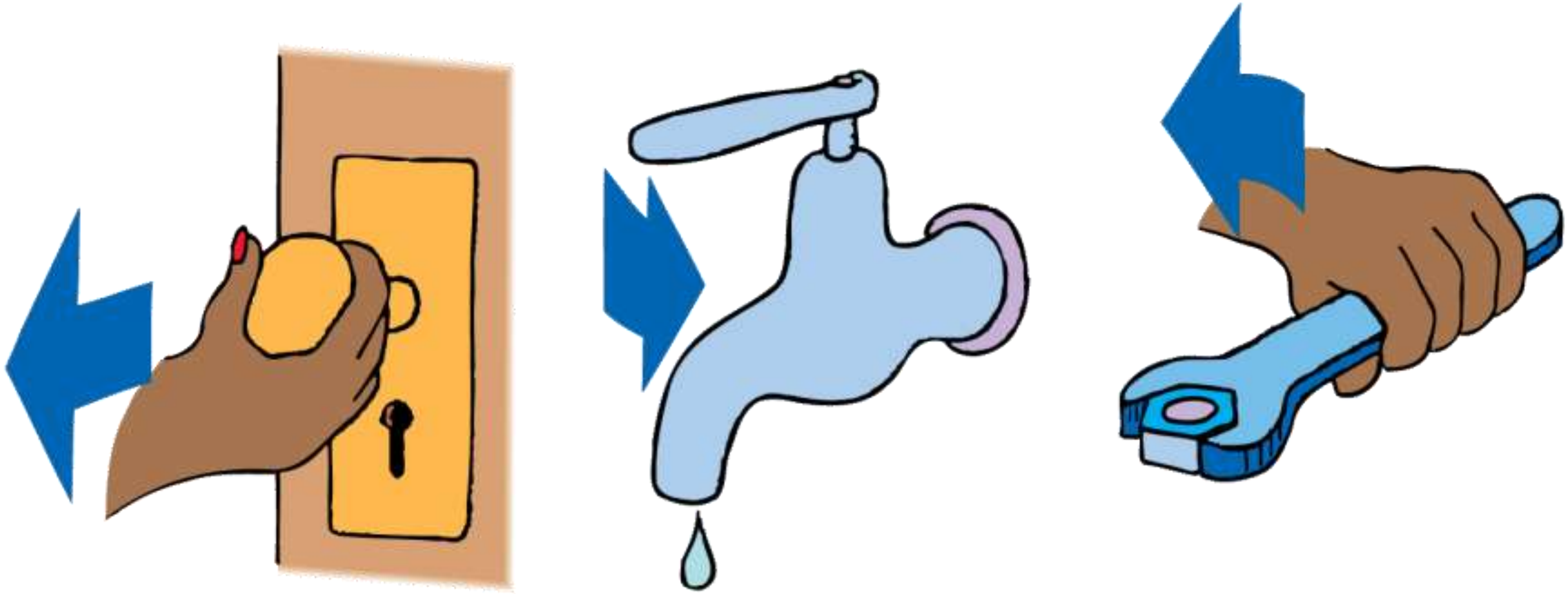
Torque is produced by this turning force and tends to produce rotational acceleration.

Torque is different from force.

- Forces tend to make things accelerate.
- Torques produce rotation.

11.1 Torque

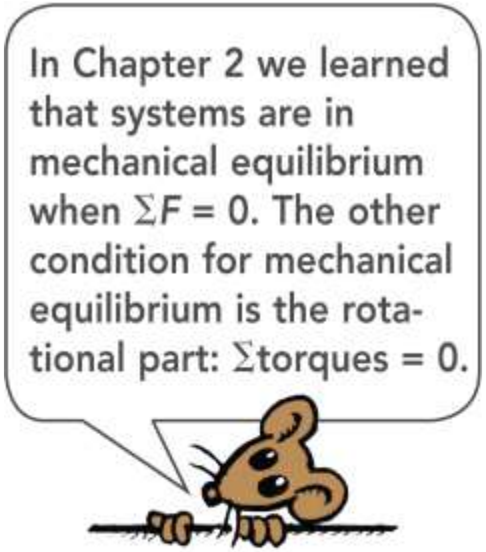
A torque produces rotation.



11.1 Torque

A torque is produced when a force is applied with "leverage."

- You use leverage when you use a claw hammer to pull a nail from a piece of wood.
- The longer the handle of the hammer, the greater the leverage and the easier the task.
- The longer handle of a crowbar provides even more leverage.



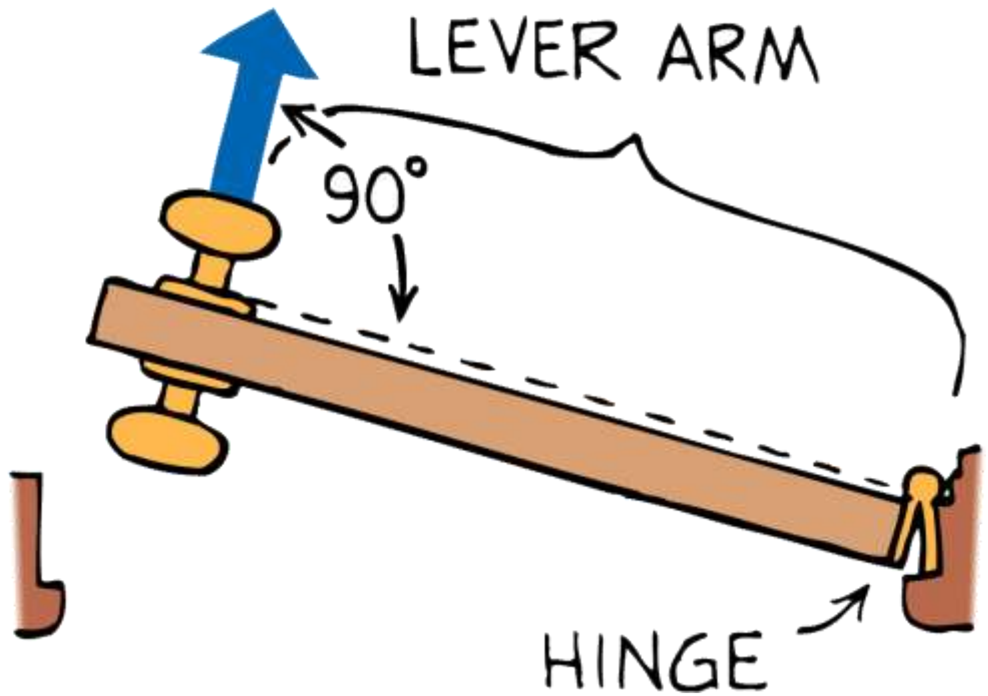
11.1 Torque

A torque is used when opening a door.

- A doorknob is placed far away from the turning axis at its hinges to provide more leverage when you push or pull on the doorknob.
- The direction of your applied force is important. In opening a door, you push *perpendicular* to the plane of the door.
- A perpendicular push or pull gives more rotation for less effort.

11.1 Torque

When a perpendicular force is applied, the lever arm is the distance between the doorknob and the edge with the hinges.



11.1 Torque

When the force is perpendicular, the distance from the turning axis to the point of contact is called the **lever arm.**

If the force is not at right angle to the lever arm, then only the perpendicular component of the force will contribute to the torque.

$$\text{torque} = \text{force}_{\perp} \times \text{lever arm}$$

11.1 Torque

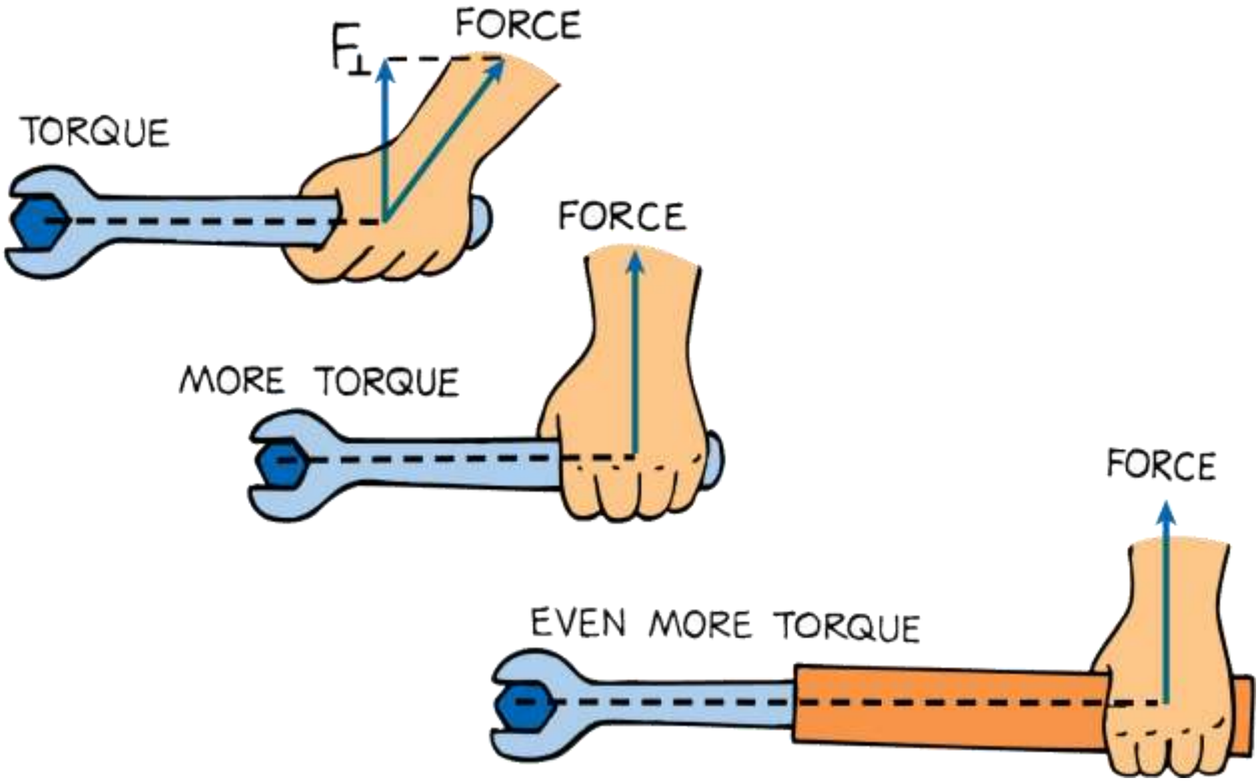
The same torque can be produced by a large force with a short lever arm, or a small force with a long lever arm.

The same force can produce different amounts of torque.

Greater torques are produced when both the force and lever arm are large.

11.1 Torque

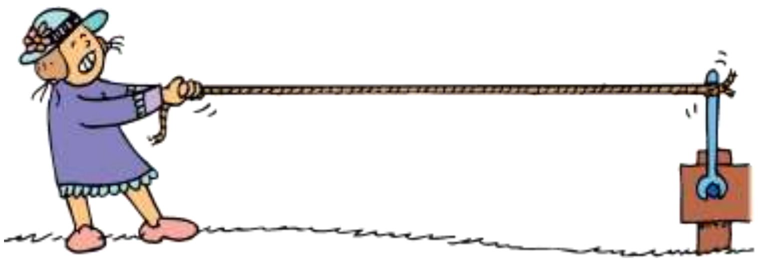
Although the magnitudes of the applied forces are the same in each case, the torques are different.



11.1 Torque

think!

If you cannot exert enough torque to turn a stubborn bolt, would more torque be produced if you fastened a length of rope to the wrench handle as shown?



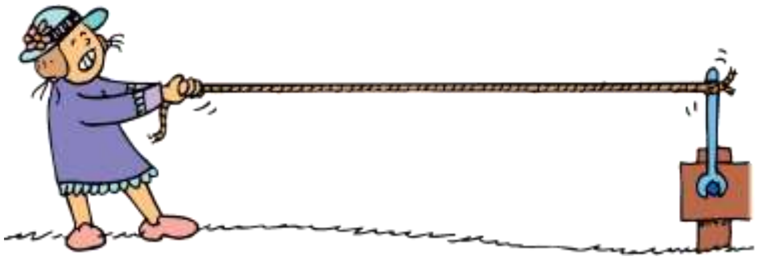
11.1 Torque

think!

If you cannot exert enough torque to turn a stubborn bolt, would more torque be produced if you fastened a length of rope to the wrench handle as shown?

Answer:

No, because the lever arm is the same. To increase the lever arm, a better idea would be to use a pipe that extends upward.



11.1 Torque

CONCEPT CHECK

How do you make an object turn or rotate?

11.2 Balanced Torques

When balanced torques act on an object, there is no change in rotation.

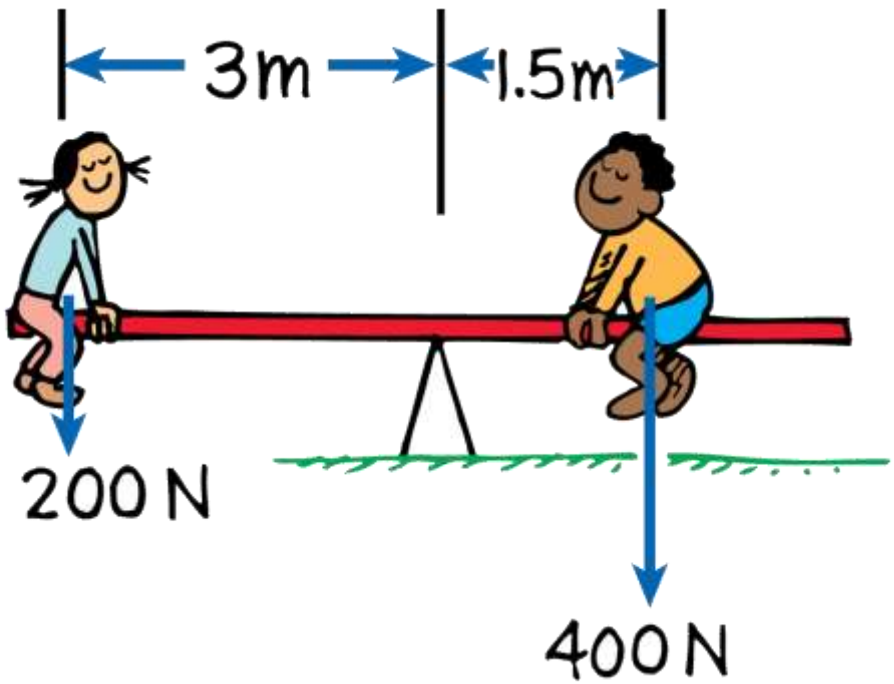
11.2 Balanced Torques

Children can balance a seesaw even when their weights are not equal.

Weight alone does not produce rotation—torque does.

11.2 Balanced Torques

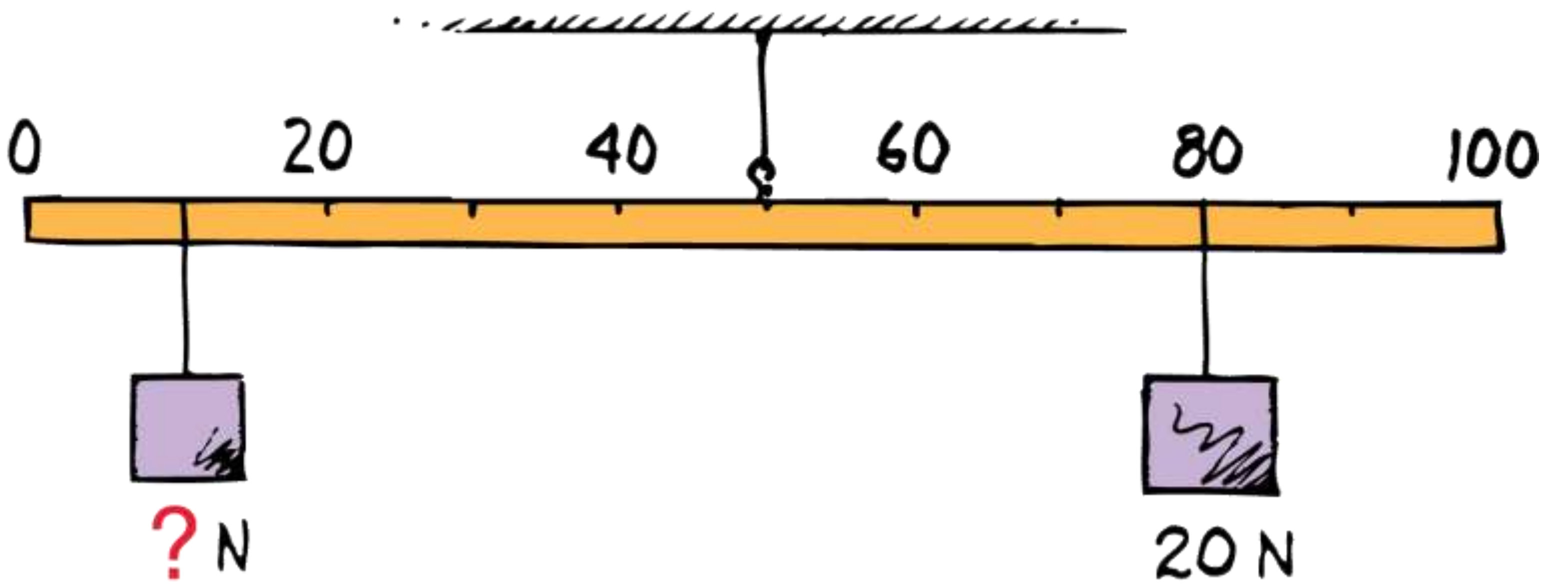
A pair of torques can balance each other. Balance is achieved if the torque that tends to produce clockwise rotation by the boy equals the torque that tends to produce counterclockwise rotation by the girl.



11.2 Balanced Torques

do the math!

What is the weight of the block hung at the 10-cm mark?



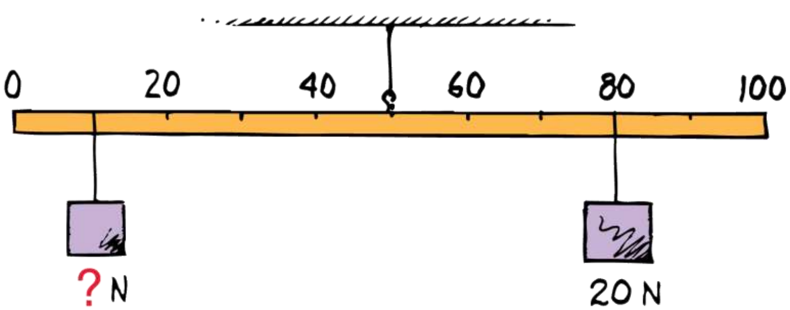
11.2 Balanced Torques

do the math!

The block of unknown weight tends to rotate the system of blocks and stick **counterclockwise**, and the 20-N block tends to rotate the system **clockwise**. The system is in balance when the two torques are equal:

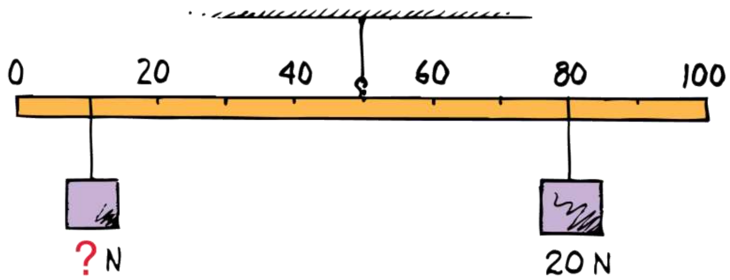
counterclockwise torque = clockwise torque

$$(F_{\perp}d)_{ccw} = (F_{\perp}d)_{cw}$$



11.2 Balanced Torques

do the math!



Rearrange the equation to solve for the unknown weight:

$$F_{\perp ccw} = \frac{(F_{\perp})_{cw} \times (d)_{cw}}{(d)_{ccw}}$$

The lever arm for the unknown weight is 40 cm.

The lever arm for the 20-N block is 30 cm.

$$F_{\perp ccw} = \frac{(20\ \text{N}) \times (30\ \text{cm})}{(40\ \text{cm})} = 15\ \text{N}$$

The unknown weight is thus 15 N.

11.2 Balanced Torques

Scale balances that work with sliding weights are based on balanced torques, not balanced masses. The sliding weights are adjusted until the counterclockwise torque just balances the clockwise torque. We say the scale is in rotational equilibrium.

11.2 Balanced Torques

CONCEPT CHECK

What happens when balanced torques act on an object?

11.3 Center of Mass

The center of mass of an object is the point located at the object’s average position of mass.

11.3 Center of Mass

A baseball thrown into the air follows a smooth parabolic path. A baseball bat thrown into the air does not follow a smooth path.

The bat wobbles about a special point. This point stays on a parabolic path, even though the rest of the bat does not.

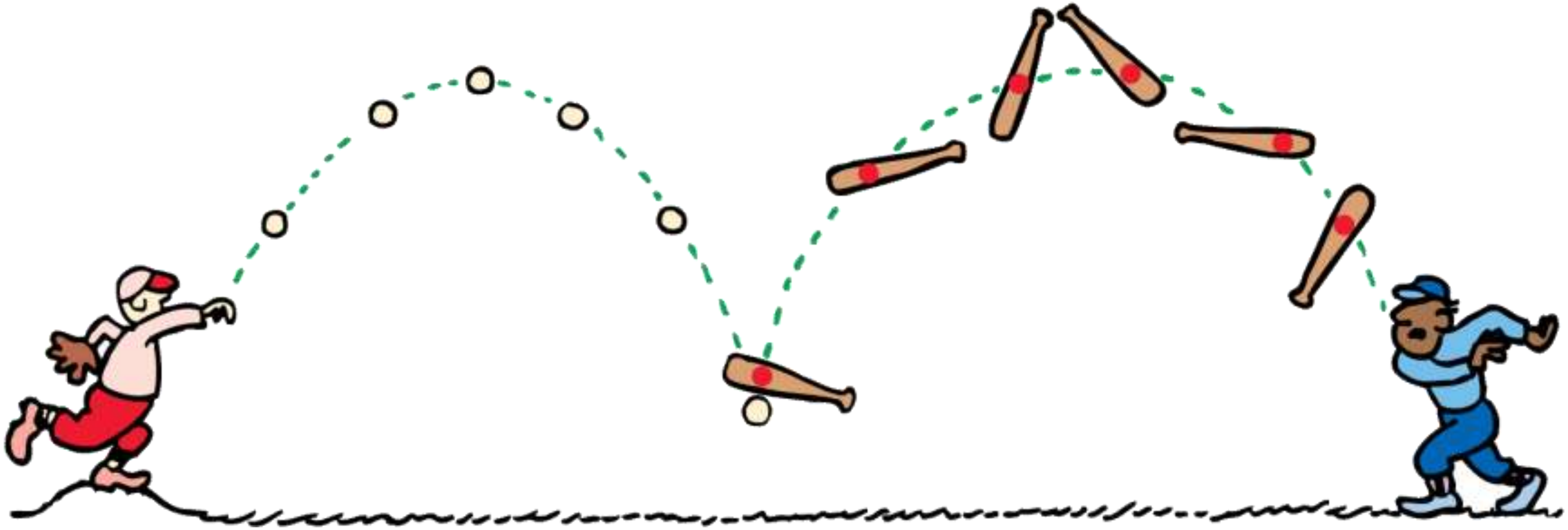
The motion of the bat is the sum of two motions:

- a spin around this point, and
- a movement through the air as if all the mass were concentrated at this point.

This point, called the **center of mass,** is where all the mass of an object can be considered to be concentrated.

11.3 Center of Mass

The centers of mass of the baseball and of the spinning baseball bat each follow parabolic paths.



11.3 Center of Mass

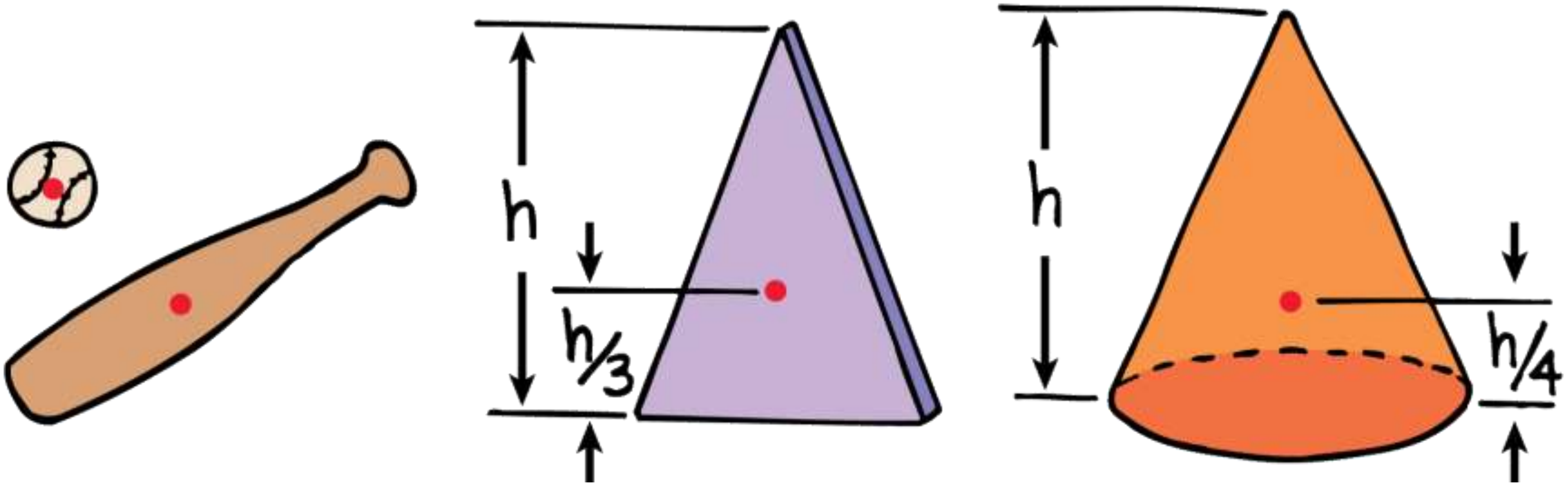
Location of the Center of Mass

For a symmetrical object, such as a baseball, the center of mass is at the geometric center of the object.

For an irregularly shaped object, such as a baseball bat, the center of mass is toward the heavier end.

11.3 Center of Mass

The center of mass for each object is shown by the red dot.



11.3 Center of Mass

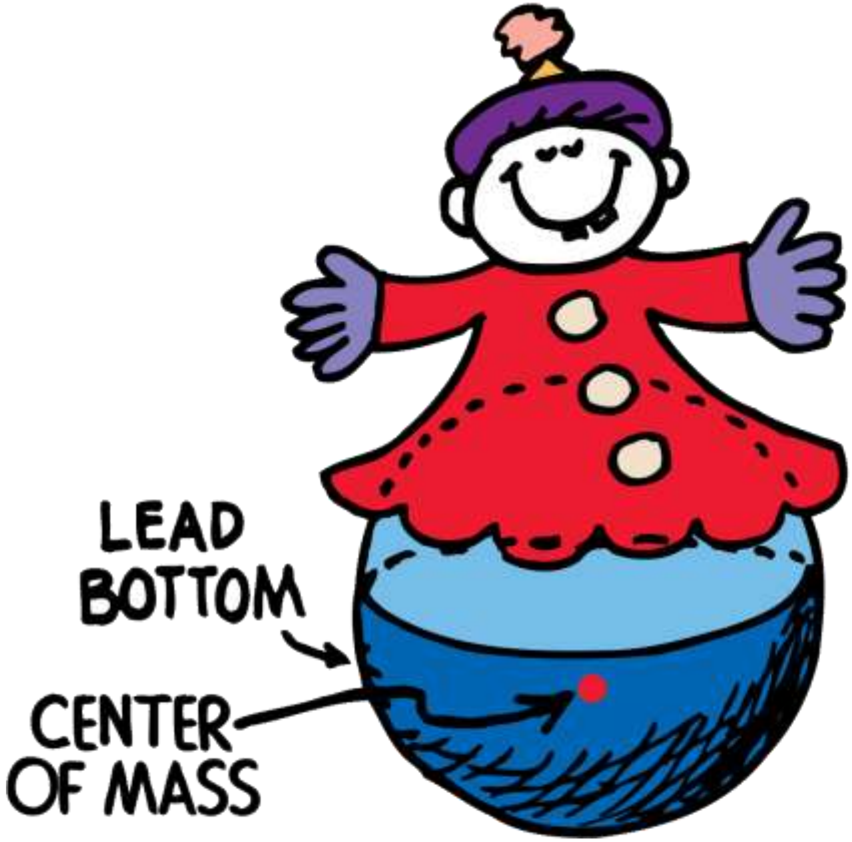
Objects not made of the same material throughout may have the center of mass quite far from the geometric center.

Consider a hollow ball half filled with lead. The center of mass would be located somewhere within the lead part.

The ball will always roll to a stop with its center of mass as low as possible.

11.3 Center of Mass

The center of mass of the toy is below its geometric center.



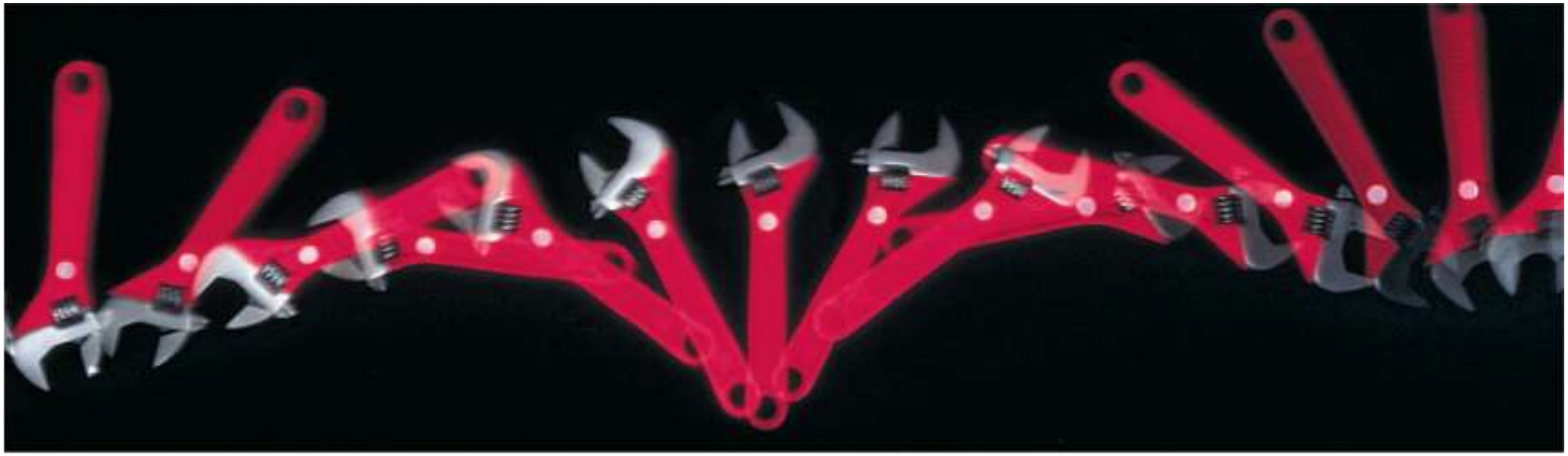
11.3 Center of Mass

Motion About the Center of Mass

As an object slides across a surface, its center of mass follows a straight-line path.

11.3 Center of Mass

The center of mass of the rotating wrench follows a straight-line path as it slides across a smooth surface.



11.3 Center of Mass

The motion of the wrench is a combination of straight-line motion of its center of mass and rotation around its center of mass.

If the wrench were tossed into the air, its center of mass would follow a smooth parabola.

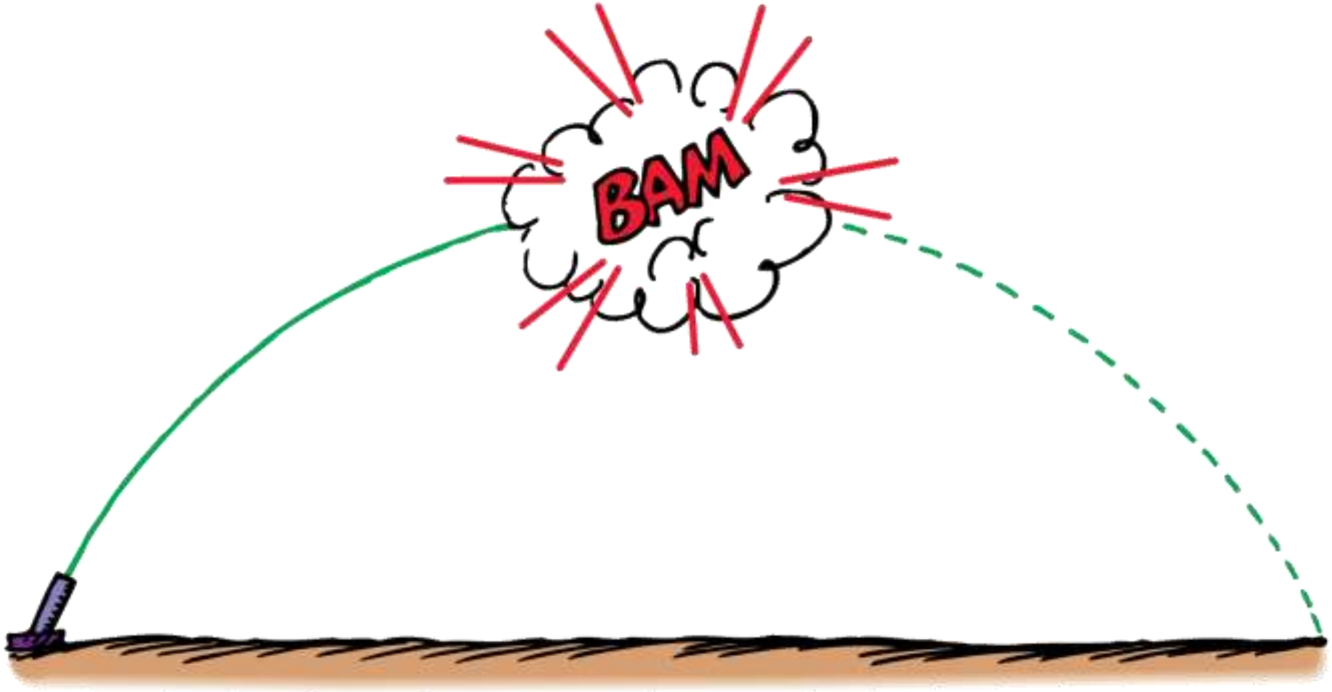
11.3 Center of Mass

Internal forces during the explosion of a projectile do not change the projectile's center of mass.

If air resistance is negligible, the center of mass of the dispersed fragments as they fly through the air will be at any time where the center of mass would have been if the explosion had never occurred.

11.3 Center of Mass

The center of mass of the fireworks rocket and its fragments move along the same path before and after the explosion.



11.3 Center of Mass

Applying Spin to an Object

When you throw a ball and apply spin to it, or when you launch a plastic flying disk, a force must be applied to the edge of the object.

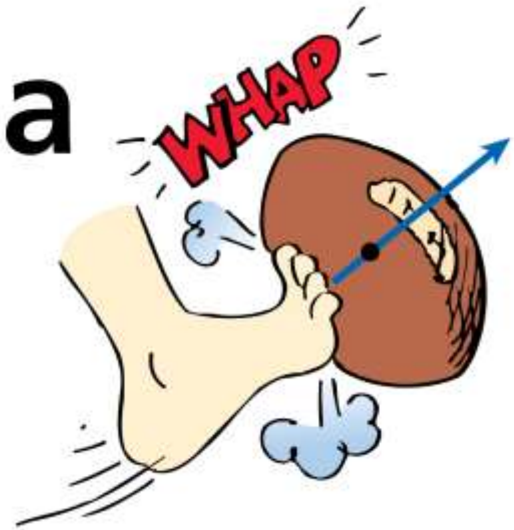
This produces a torque that adds rotation to the projectile.

A skilled pool player strikes the cue ball below its center to put backspin on the ball.

11.3 Center of Mass

A force must be applied to the edge of an object for it to spin.

a. If the football is kicked in line with its center, it will move without rotating.

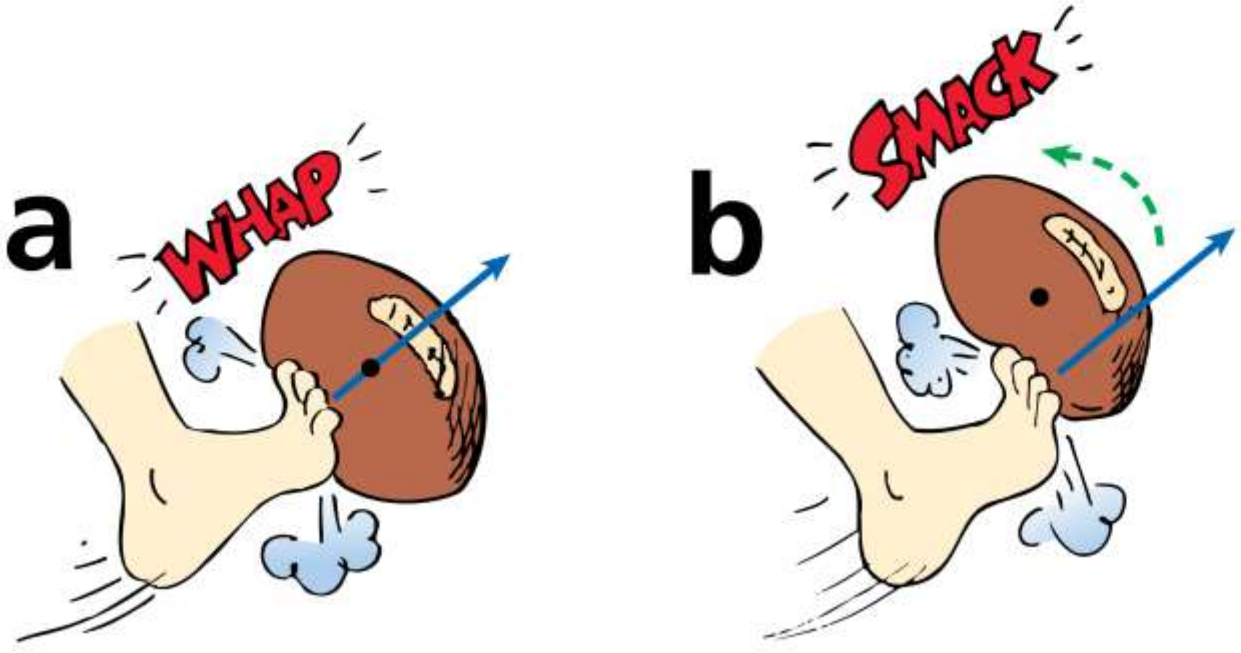


11.3 Center of Mass

A force must be applied to the edge of an object for it to spin.

a. If the football is kicked in line with its center, it will move without rotating.

b. If it is kicked above or below its center, it will rotate.



11.3 Center of Mass

CONCEPT CHECK

Where is an object's center of mass located?

11.4 Center of Gravity

For everyday objects, the center of gravity is the same as the center of mass.

11.4 Center of Gravity

Center of mass is often called **center of gravity,** the average position of all the particles of *weight* that make up an object.

For almost all objects on and near Earth, these terms are interchangeable.

There can be a small difference between center of gravity and center of mass when an object is large enough for gravity to vary from one part to another.

The center of gravity of the Sears Tower in Chicago is about 1 mm below its center of mass because the lower stories are pulled a little more strongly by Earth's gravity than the upper stories.

11.4 Center of Gravity

Wobbling

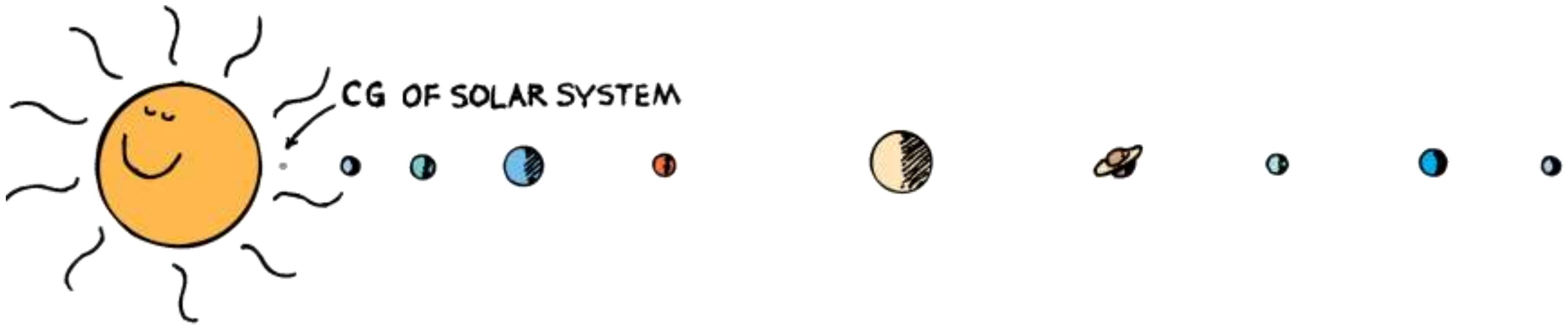
If you threw a wrench so that it rotated as it moved through the air, you'd see it wobble about its center of gravity. The center of gravity itself would follow a parabolic path.

The sun itself wobbles off-center.

- As the planets orbit the sun, the center of gravity of the solar system can lie outside the massive sun.
- Astronomers look for similar wobbles in nearby stars—the wobble is an indication of a star with a planetary system.

11.4 Center of Gravity

If all the planets were lined up on one side of the sun, the center of gravity of the solar system would lie outside the sun.



11.4 Center of Gravity

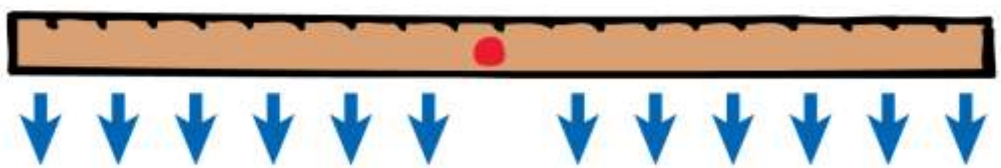
Locating the Center of Gravity

The center of gravity (CG) of a uniform object is at the midpoint, its geometric center.

- The CG is the balance point.
- Supporting that single point supports the whole object.

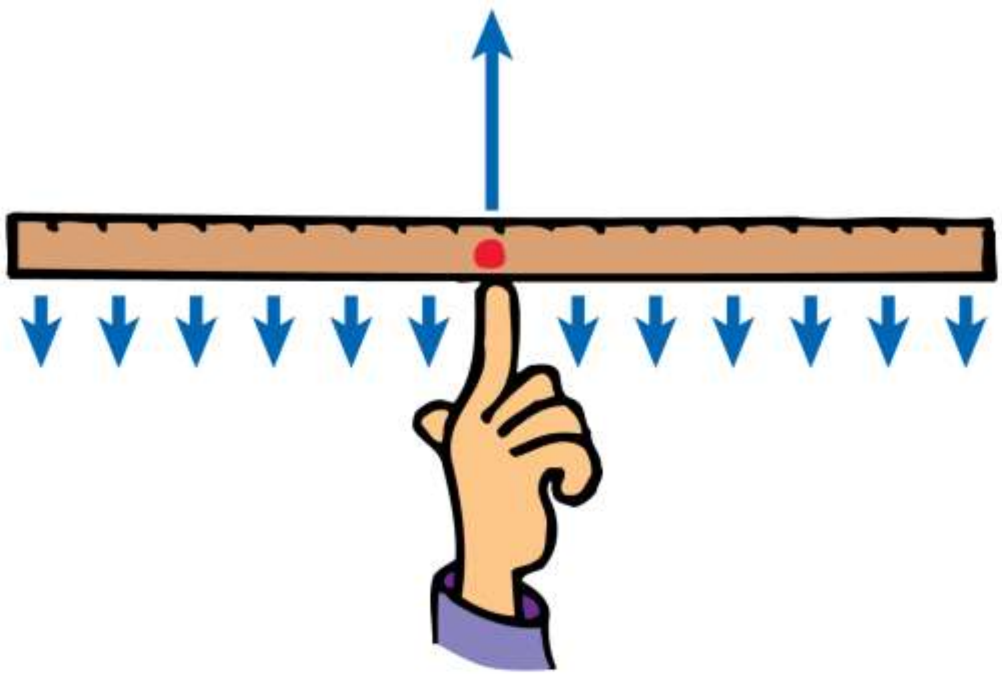
11.4 Center of Gravity

The weight of the entire stick behaves as if it were concentrated at its center. The small vectors represent the force of gravity along the meter stick, which combine into a resultant force that acts at the CG.



11.4 Center of Gravity

The weight of the entire stick behaves as if it were concentrated at its center. The small vectors represent the force of gravity along the meter stick, which combine into a resultant force that acts at the CG.



11.4 Center of Gravity

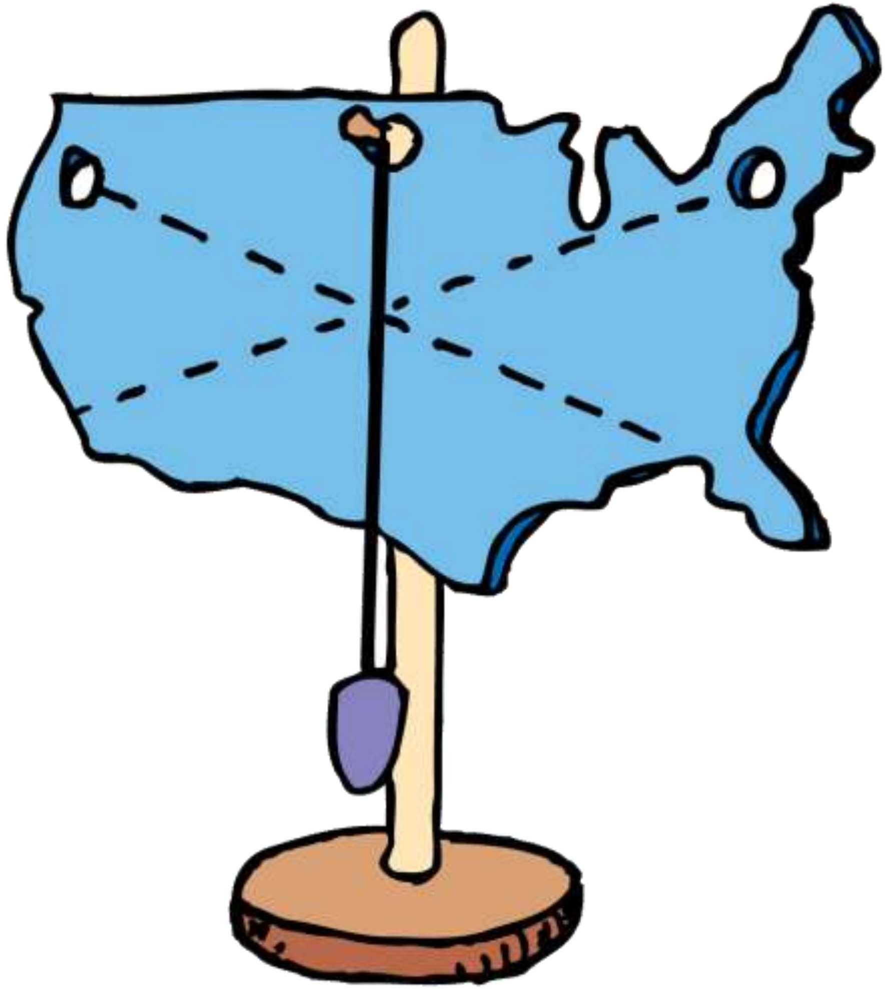
If you suspend any object at a single point, the CG of the object will hang directly below (or at) the point of suspension.

To locate an object's CG:

- Construct a vertical line beneath the point of suspension.
- The CG lies somewhere along that line.
- Suspend the object from some other point and construct a second vertical line.
- The CG is where the two lines intersect.

11.4 Center of Gravity

You can use a plumb bob to find the CG for an irregularly shaped object.



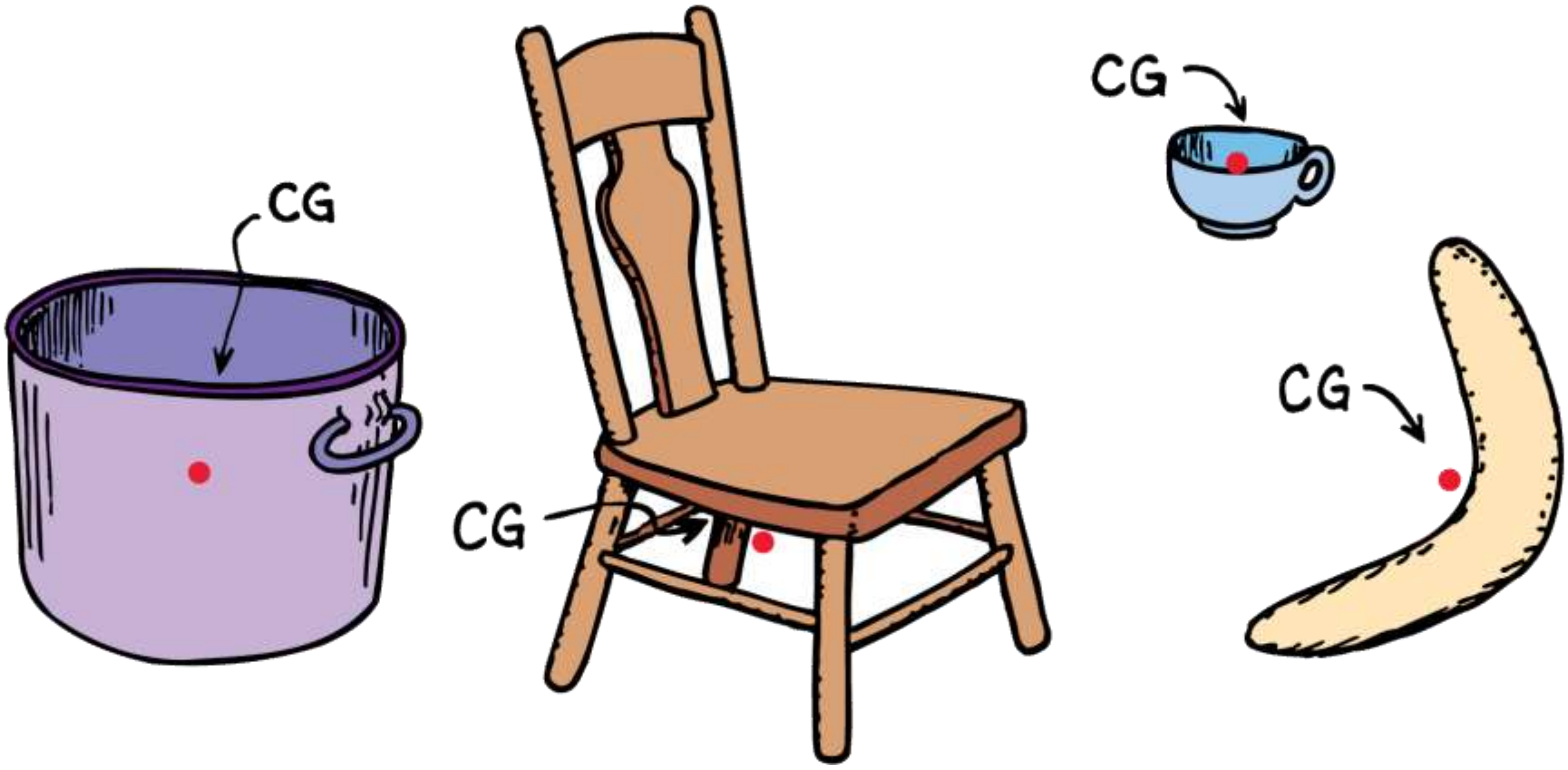
11.4 Center of Gravity

The CG of an object may be located where no actual material exists.

- The CG of a ring lies at the geometric center where no matter exists.
- The same holds true for a hollow sphere such as a basketball.

11.4 Center of Gravity

There is no material at the CG of these objects.



11.4 Center of Gravity

think!

Where is the CG of a donut?

11.4 Center of Gravity

think!

Where is the CG of a donut?

Answer:

In the center of the hole!

11.4 Center of Gravity

think!

Can an object have more than one CG?

11.4 Center of Gravity

think!

Can an object have more than one CG?

Answer:

No. A rigid object has one CG. If it is nonrigid, such as a piece of clay or putty, and is distorted into different shapes, then its CG may change as its shape is changed. Even then, it has one CG for any given shape.

11.4 Center of Gravity

CONCEPT CHECK

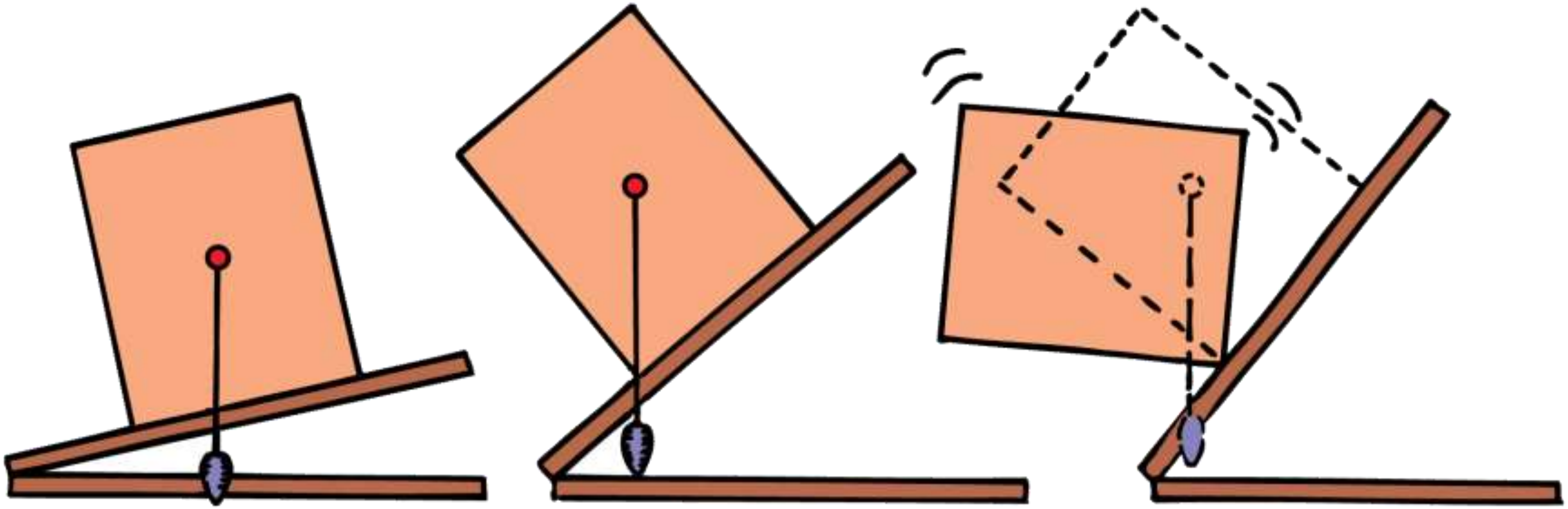
How is the center of gravity of an everyday object related to its center of mass?

11.5 Torque and Center of Gravity

If the center of gravity of an object is above the area of support, the object will remain upright.

11.5 Torque and Center of Gravity

The block topples when the CG extends beyond its support base.



11.5 Torque and Center of Gravity

The Rule for Toppling

If the CG extends outside the area of support, an unbalanced torque exists, and the object will topple.

11.5 Torque and Center of Gravity

This "Londoner" double-decker bus is undergoing a tilt test.

So much of the weight of the vehicle is in the lower part that the bus can be tilted beyond 28° without toppling.

11.5 Torque and Center of Gravity

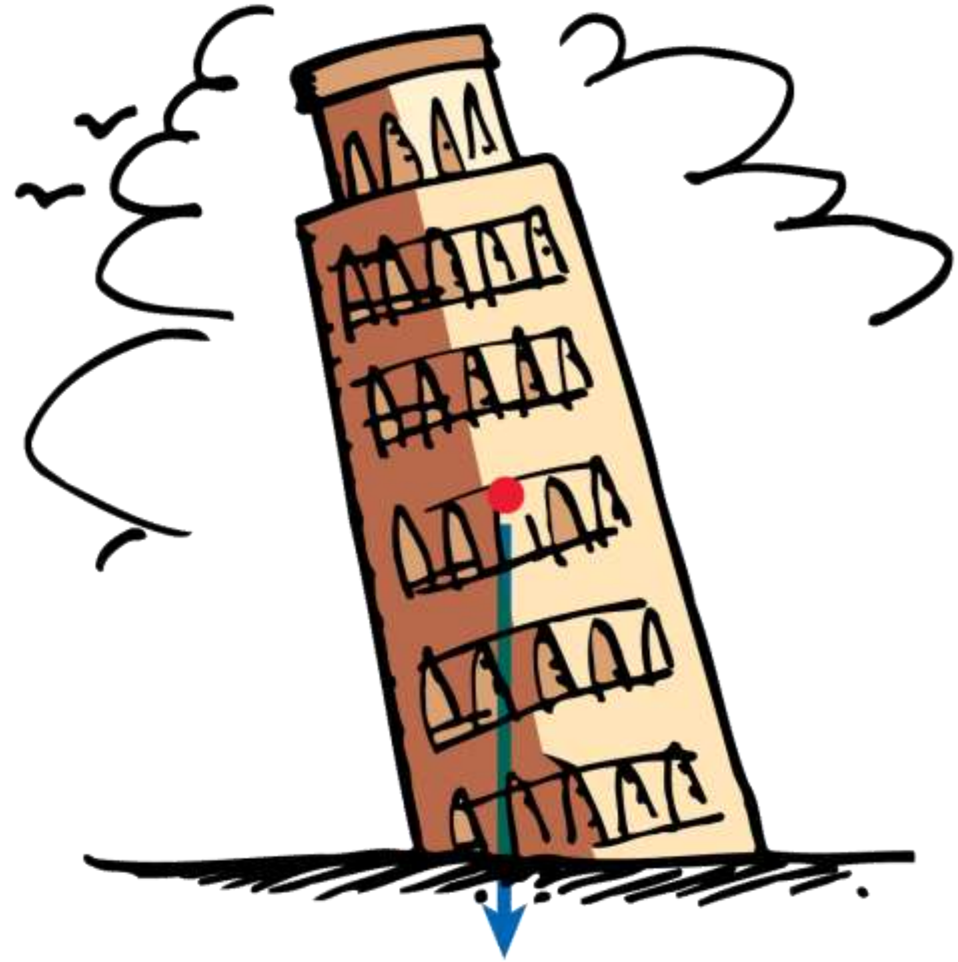
The Leaning Tower of Pisa does not topple because its CG does not extend beyond its base.

A vertical line below the CG falls inside the base, and so the Leaning Tower has stood for centuries.

If the tower leaned far enough that the CG extended beyond the base, an unbalanced torque would topple the tower.

11.5 Torque and Center of Gravity

The Leaning Tower of Pisa does not topple over because its CG lies above its base.



11.5 Torque and Center of Gravity

The support base of an object does not have to be solid.

An object will remain upright if the CG is above its base of support.

11.5 Torque and Center of Gravity

The shaded area bounded by the bottom of the chair legs defines the support base of the chair.

11.5 Torque and Center of Gravity

Balancing

Try balancing a broom upright on the palm of your hand.

The support base is quite small and relatively far beneath the CG, so it's difficult to maintain balance for very long.

After some practice, you can do it if you learn to make slight movements of your hand to exactly respond to variations in balance.

11.5 Torque and Center of Gravity

Gyroscopes and computer-assisted motors in the self-balancing electric scooter make continual adjustments to keep the combined CGs of Mark, Tenny, and the vehicles above the support base.

11.5 Torque and Center of Gravity

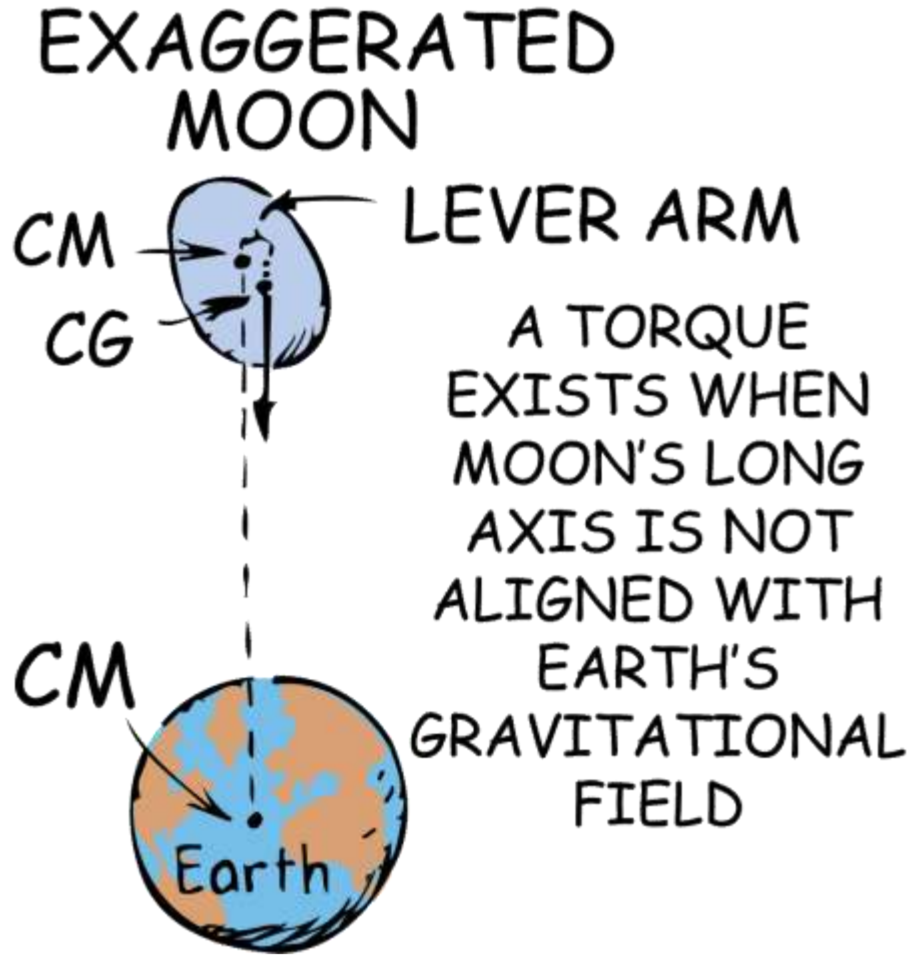
The Moon's CG

Only one side of the moon continually faces Earth.

- Because the side of the moon nearest Earth is gravitationally tugged toward Earth a bit more than farther parts, the moon's CG is closer to Earth than its center of mass.
- While the moon rotates about its center of mass, Earth pulls on its CG.
- This produces a torque when the moon's CG is not on the line between the moon's and Earth's centers.
- This torque keeps one hemisphere of the moon facing Earth.

11.5 Torque and Center of Gravity

The moon is slightly football-shaped due to Earth's gravitational pull.



11.5 Torque and Center of Gravity

CONCEPT CHECK

What is the rule for toppling?

11.6 Center of Gravity of People

The center of gravity of a person is not located in a fixed place, but depends on body orientation.

11.6 Center of Gravity of People

When you stand erect with your arms hanging at your sides, your CG is within your body, typically 2 to 3 cm below your navel, and midway between your front and back.

Raise your arms vertically overhead. Your CG rises 5 to 8 cm.

Bend your body into a U or C shape and your CG may be located outside your body altogether.

11.6 Center of Gravity of People

A high jumper executes a “Fosbury flop” to clear the bar while his CG nearly passes beneath the bar.

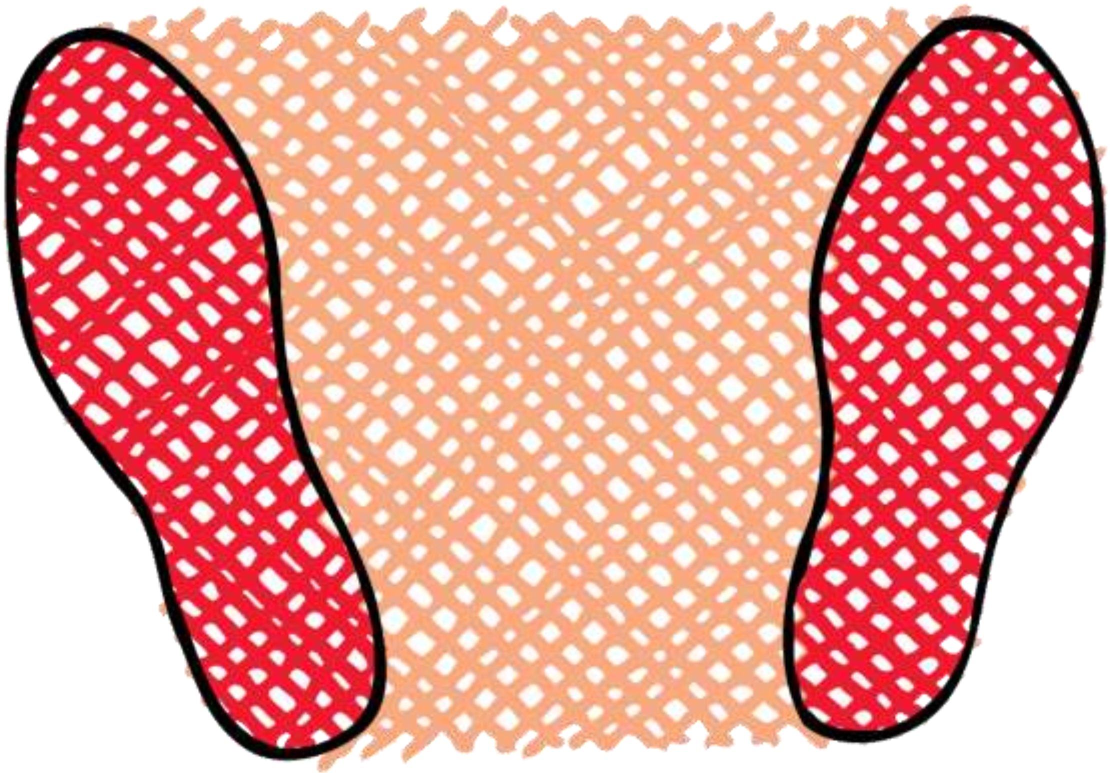
11.6 Center of Gravity of People

When you stand, your CG is somewhere above your support base, the area bounded by your feet.

- In unstable situations, as in standing in the aisle of a bumpy-riding bus, you place your feet farther apart to increase this area.
- Standing on one foot greatly decreases this area.
- In learning to walk, a baby must learn to coordinate and position the CG above a supporting foot.

11.6 Center of Gravity of People

When you stand, your CG is somewhere above the area bounded by your feet.



11.6 Center of Gravity of People

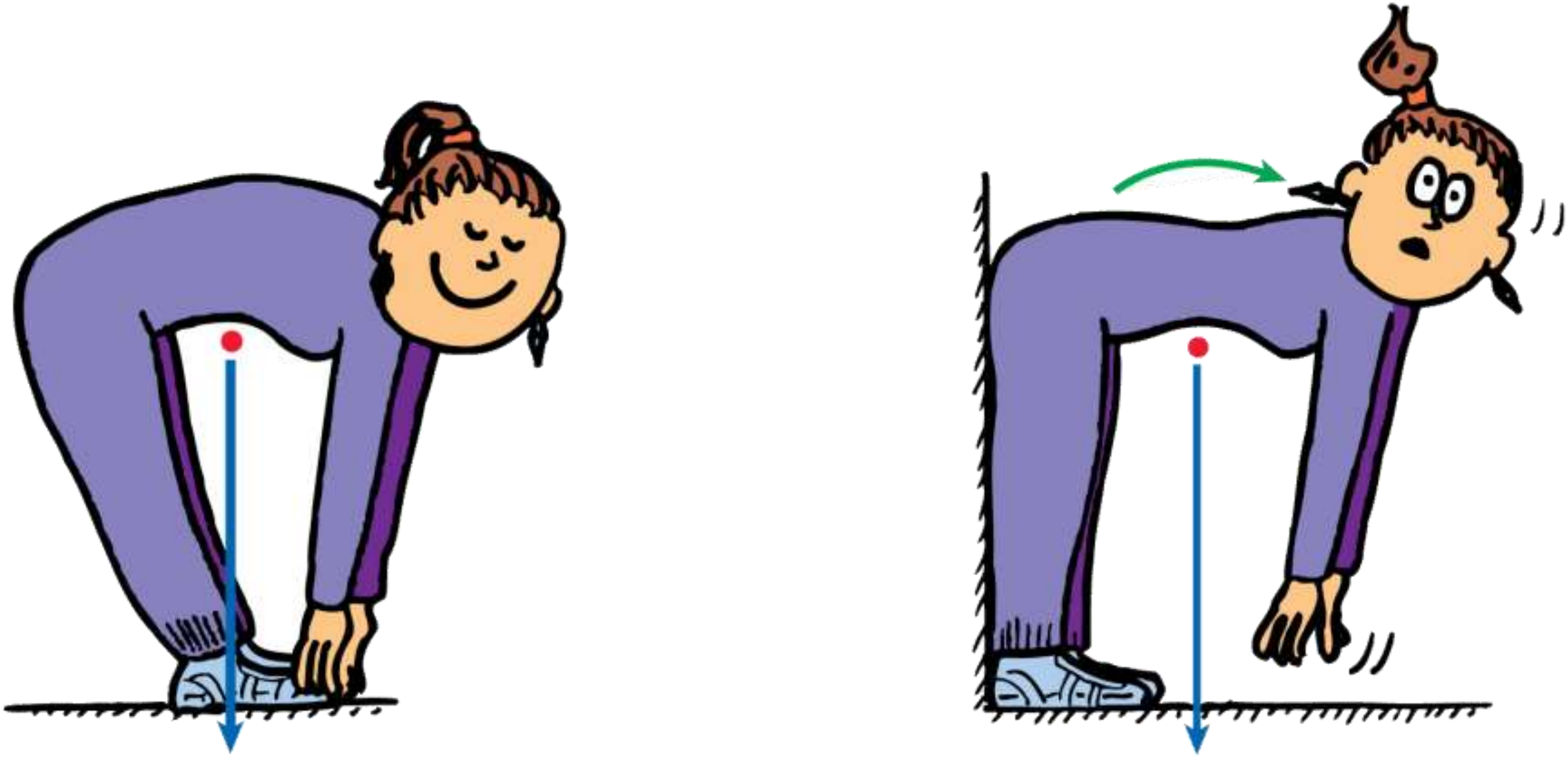
You can probably bend over and touch your toes without bending your knees.

In doing so, you unconsciously extend the lower part of your body so that your CG, which is now outside your body, is still above your supporting feet.

Try it while standing with your heels to a wall. You are unable to adjust your body, and your CG protrudes beyond your feet. You are off balance and torque topples you over.

11.6 Center of Gravity of People

You can lean over and touch your toes without toppling only if your CG is above the area bounded by your feet.



11.6 Center of Gravity of People

think!

When you carry a heavy load—such as a pail of water—with one arm, why do you tend to hold your free arm out horizontally?

11.6 Center of Gravity of People

think!

When you carry a heavy load—such as a pail of water—with one arm, why do you tend to hold your free arm out horizontally?

Answer:

You tend to hold your free arm outstretched to shift the CG of your body away from the load so your combined CG will more easily be above the base of support. To really help matters, divide the load in two if possible, and carry half in each hand. Or, carry the load on your head!

11.6 Center of Gravity of People

CONCEPT CHECK

On what does the location of a person's center of gravity depend?

11.7 Stability

When an object is toppled, the center of gravity of that object is raised, lowered, or unchanged.

11.7 Stability

It is nearly impossible to balance a pen upright on its point, while it is rather easy to stand it upright on its flat end.

- The base of support is inadequate for the point and adequate for the flat end.
- Also, even if you position the pen so that its CG is exactly above its tip, the slightest vibration or air current can cause it to topple.

11.7 Stability

Change in the Location of the CG Upon Toppling

What happens to the CG of a cone standing on its point when it topples?

The CG is lowered by *any* movement.

We say that an object balanced so that any displacement lowers its center of mass is in **unstable equilibrium.**

11.7 Stability

A cone balances easily on its base.

To make it topple, its CG must be raised.

This means the cone's potential energy must be increased, which requires work.

We say an object that is balanced so that any displacement raises its center of mass is in **stable equilibrium.**

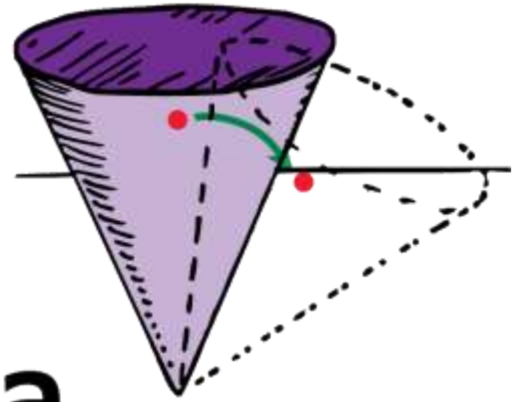
11.7 Stability

A cone on lying on its side is balanced so that any small movement neither raises nor lowers its center of gravity.

The cone is in **neutral equilibrium.**

11.7 Stability

a. Equilibrium is *unstable* when the CG is lowered with displacement.



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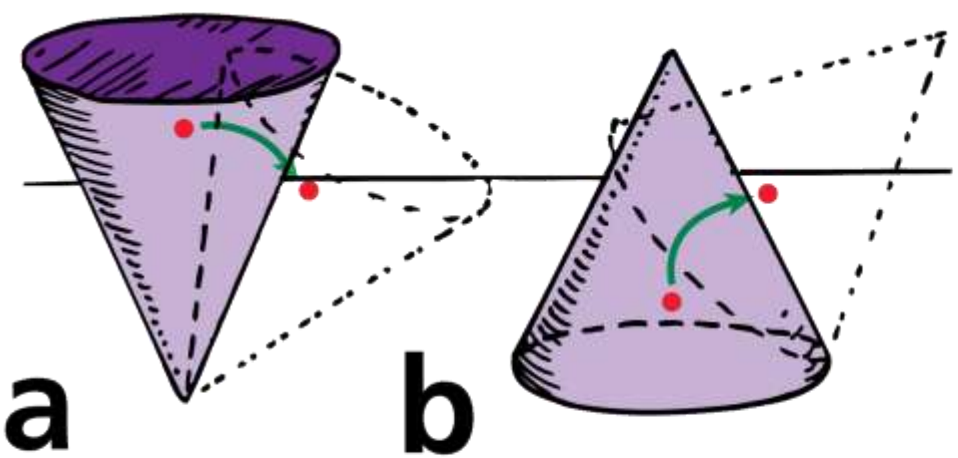
11.7 Stability

a. Equilibrium is *unstable* when the CG is lowered with displacement.

b. Equilibrium is *stable* when work must be done to raise the CG.

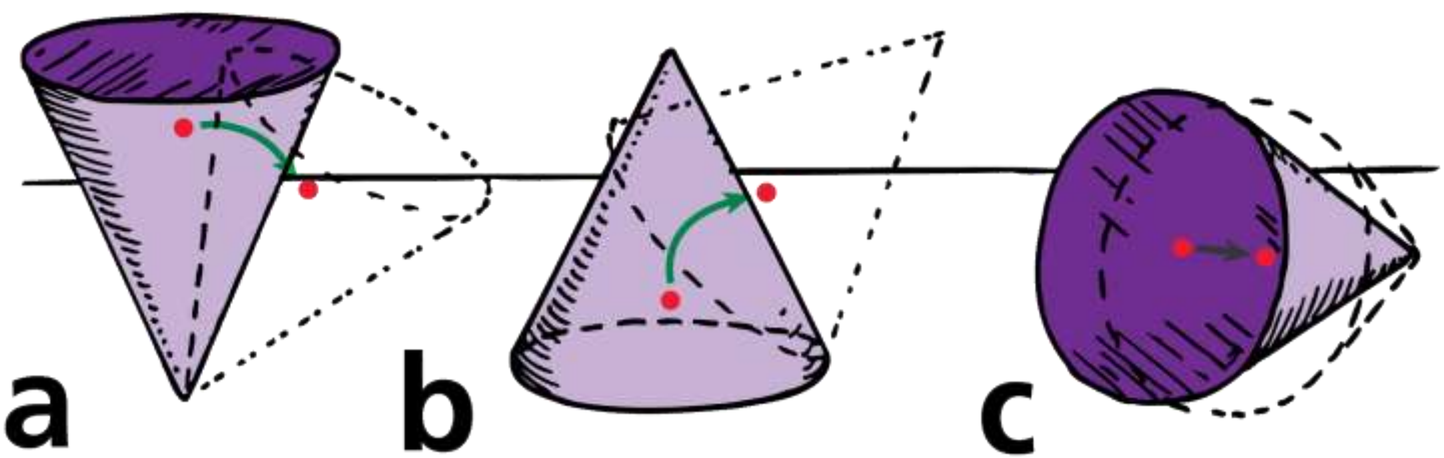


11.7 Stability

a. Equilibrium is *unstable* when the CG is lowered with displacement.

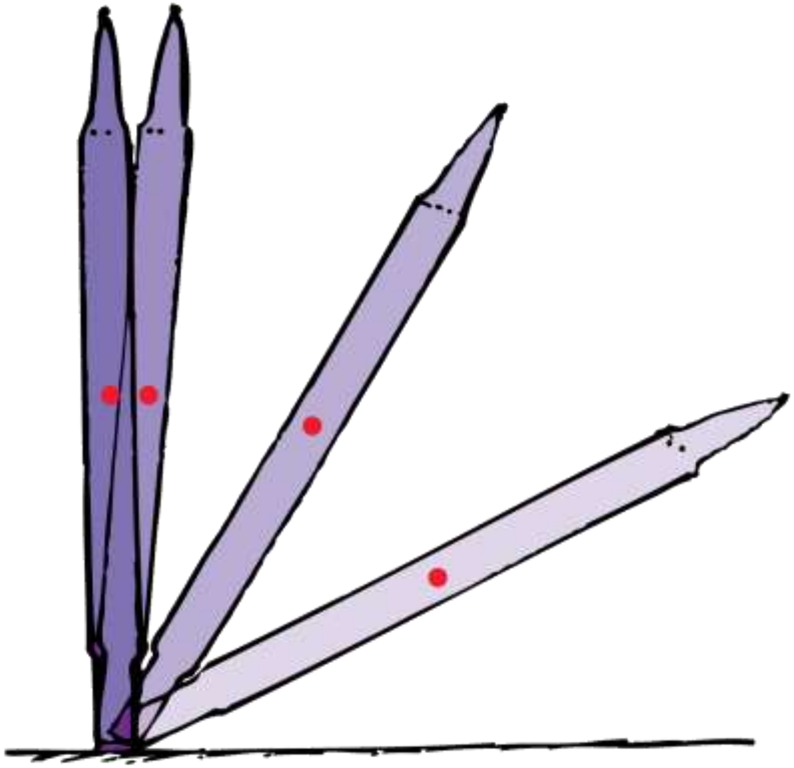
b. Equilibrium is *stable* when work must be done to raise the CG.

c. Equilibrium is *neutral* when displacement neither raises nor lowers the CG.



11.7 Stability

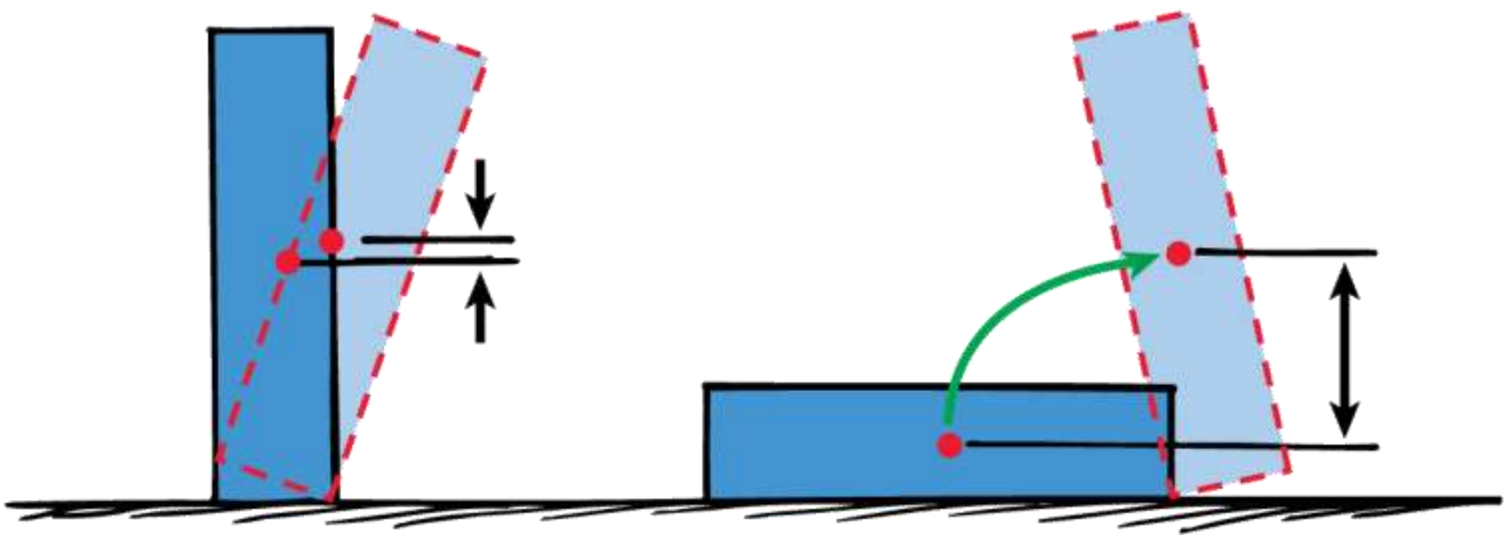
For the pen to topple when it is on its flat end, it must rotate over one edge. During the rotation, the CG rises slightly and then falls.



11.7 Stability

Toppling the upright book requires only a slight raising of its CG. Toppling the flat book requires a relatively large raising of its CG.

An object with a low CG is usually more stable than an object with a relatively high CG.



11.7 Stability

Objects in Stable Equilibrium

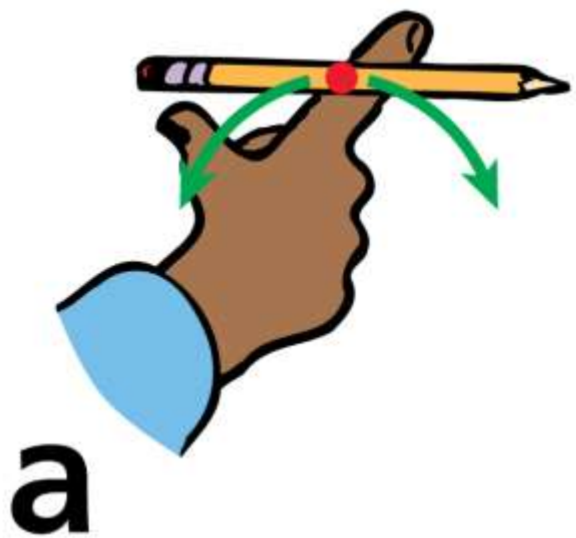
The horizontally balanced pencil is in unstable equilibrium. Its CG is lowered when it tilts.

But suspend a potato from each end and the pencil becomes stable because the CG is below the point of support, and is raised when the pencil is tilted.

11.7 Stability

A pencil balanced on the edge of a hand is in unstable equilibrium.

a. The CG of the pencil is lowered when it tilts.

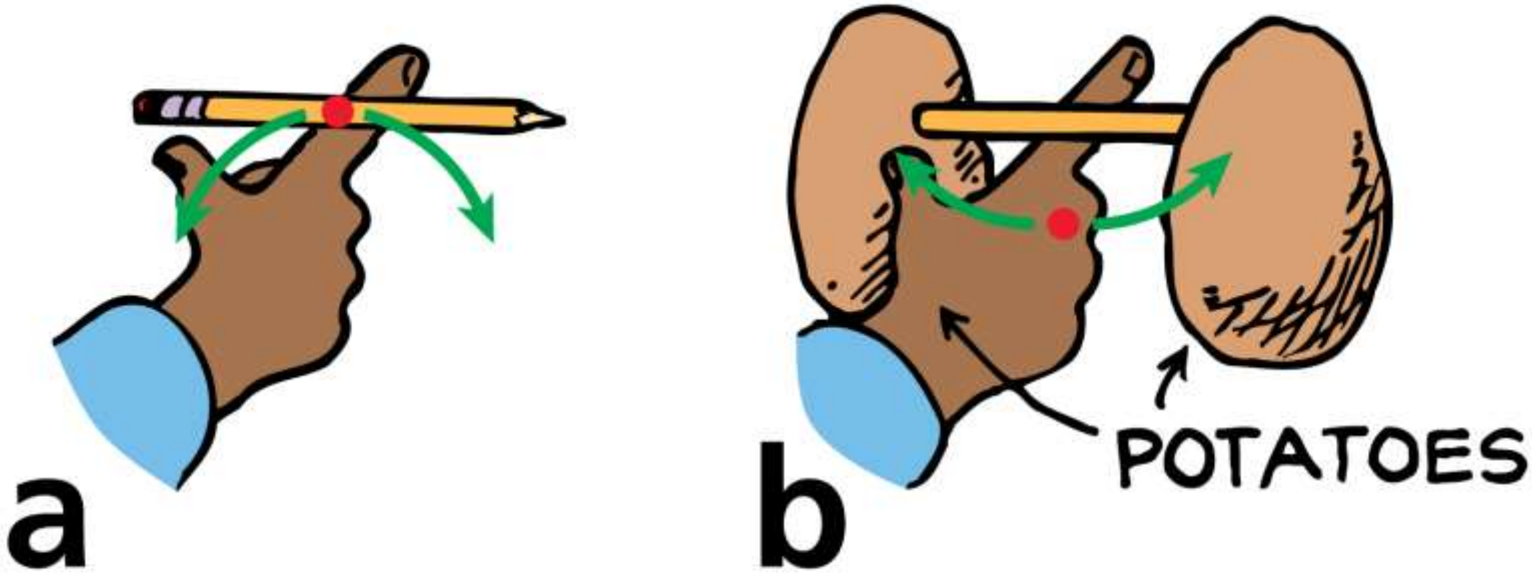


11.7 Stability

A pencil balanced on the edge of a hand is in unstable equilibrium.

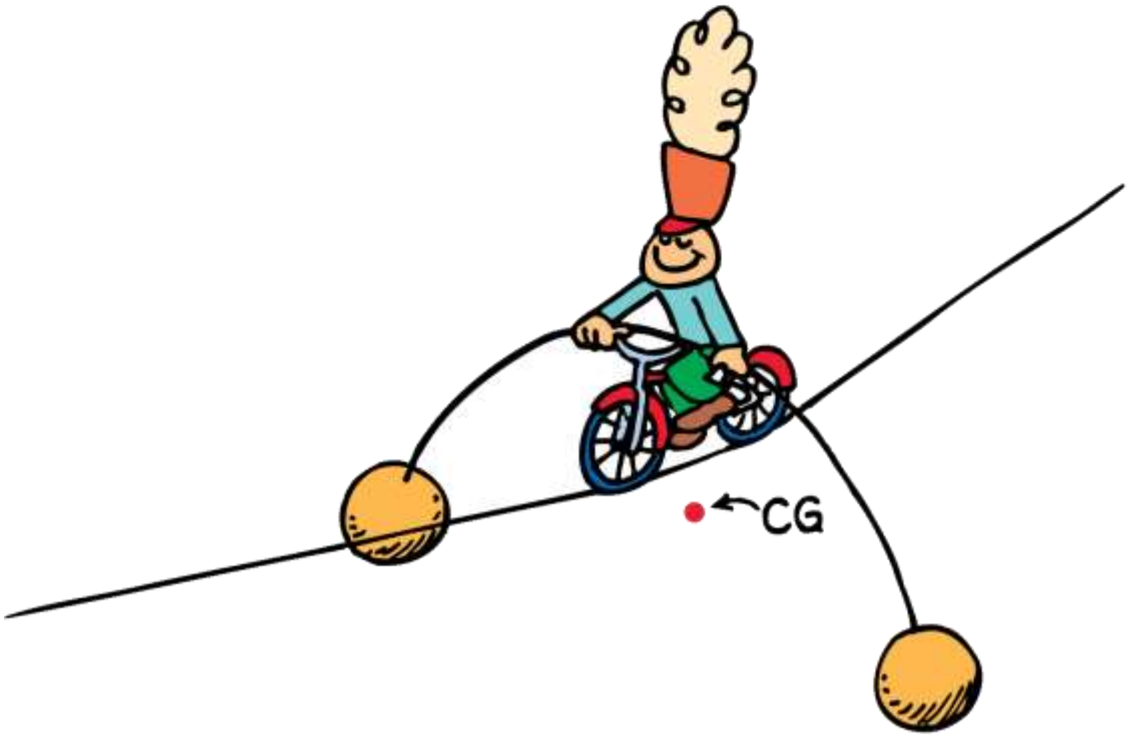
a. The CG of the pencil is lowered when it tilts.

b. When the ends of the pencil are stuck into long potatoes that hang below, it is stable because its CG rises when it is tipped.



11.7 Stability

The toy is in stable equilibrium because the CG rises when the toy tilts.



11.7 Stability

The CG of a building is lowered if much of the structure is below ground level.

This is important for tall, narrow structures.

11.7 Stability

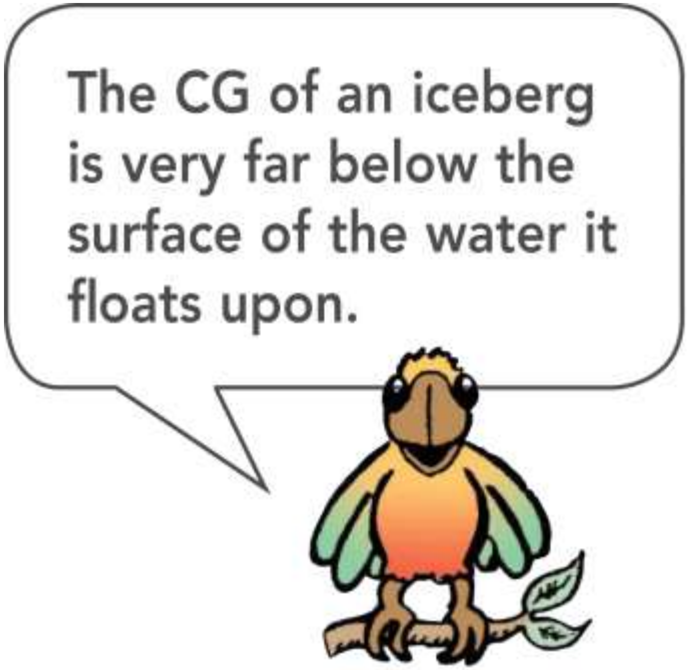
The Seattle Space Needle is so “deeply rooted” that its center of mass is actually below ground level.

It cannot fall over intact because falling would not lower its CG at all. If the structure were to tilt intact onto the ground, its CG would be raised!

11.7 Stability

Lowering the CG of an Object

The CG of an object tends to take the lowest position available.



11.7 Stability

The CG of an object has a tendency to take the lowest position available.

a. A table tennis ball is placed at the bottom of a container of dried beans.

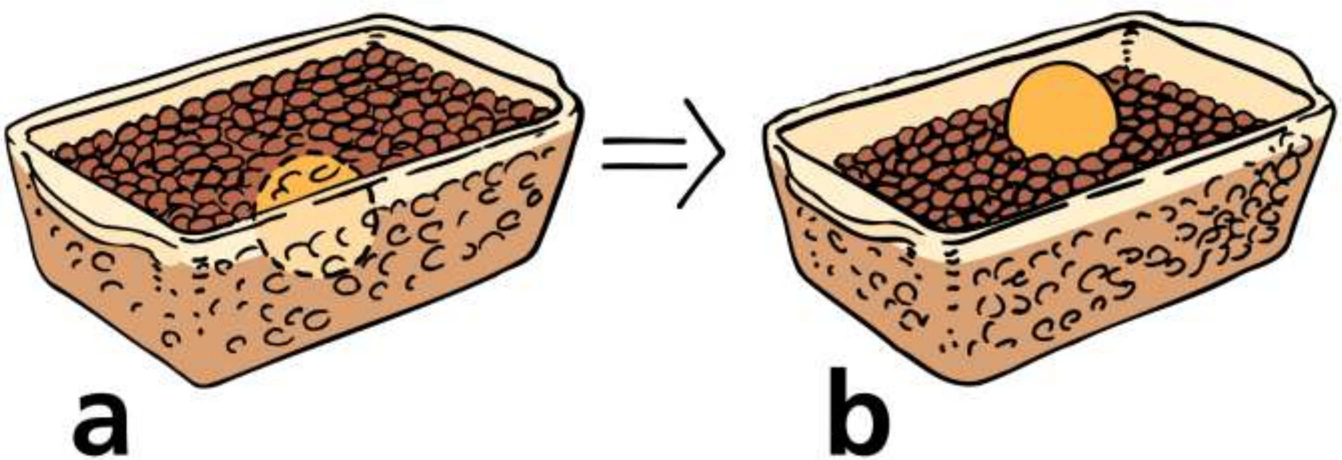
a

11.7 Stability

The CG of an object has a tendency to take the lowest position available.

a. A table tennis ball is placed at the bottom of a container of dried beans.

b. When the container is shaken from side to side, the ball is nudged to the top.



11.7 Stability

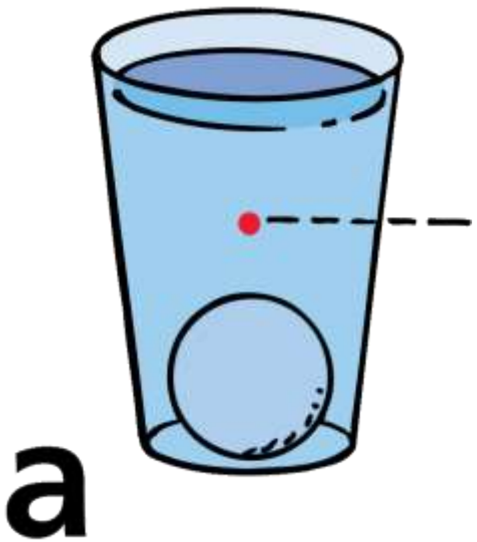
The same thing happens when an object is placed in water:

- If the object weighs less than an equal volume of water, the object is forced to the surface. The CG of the whole system will be lowered because the heavier water occupies the lower space.
- If the object is heavier than an equal volume of water, it will be more dense than water and sink. The CG of the whole system is lowered.
- If the object weighs the same as an equal volume of water, the CG of the system is unchanged whether the object rises or sinks.

11.7 Stability

The CG of the glass of water is affected by the position of the table tennis ball.

a. The CG is higher when the ball is anchored to the bottom.

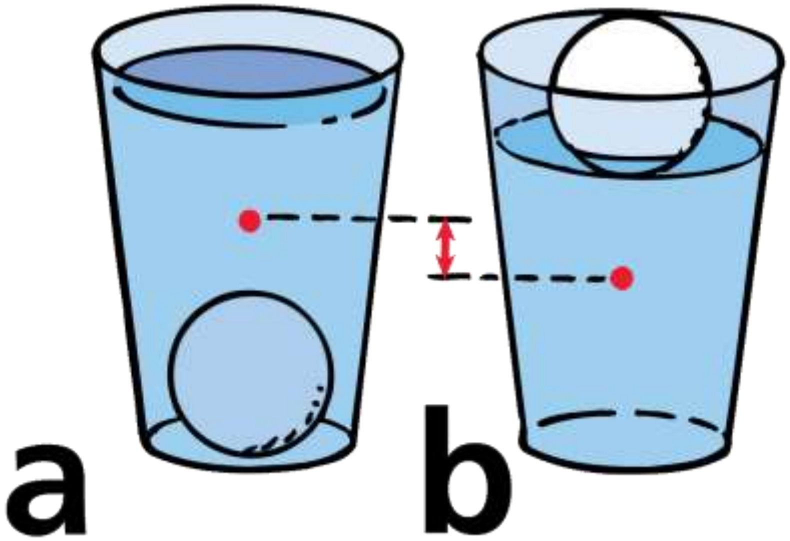


11.7 Stability

The CG of the glass of water is affected by the position of the table tennis ball.

a. The CG is higher when the ball is anchored to the bottom.

b. The CG is lower when the ball floats.



11.7 Stability

CONCEPT CHECK

What happens to the center of gravity when an object is toppled?

Assessment Questions

1. Applying a longer lever arm to an object so it will rotate produces
    a. less torque.
    b. more torque.
    c. less acceleration.
    d. more acceleration.

Assessment Questions

1. Applying a longer lever arm to an object so it will rotate produces
    a. less torque.
    b. more torque.
    c. less acceleration.
    d. more acceleration.

Answer: B

Assessment Questions

2. When two children of different weights balance on a seesaw, they each produce
   a. equal torques in the same direction.
   b. unequal torques.
   c. equal torques in opposite directions.
   d. equal forces.

Assessment Questions

2. When two children of different weights balance on a seesaw, they each produce
   a. equal torques in the same direction.
   b. unequal torques.
   c. equal torques in opposite directions.
   d. equal forces.

Answer: C

Assessment Questions

3. The center of mass of a donut is located
   a. in the hole.
   b. in material making up the donut.
   c. near the center of gravity.
   d. over a point of support.

Assessment Questions

3. The center of mass of a donut is located
   a. in the hole.
   b. in material making up the donut.
   c. near the center of gravity.
   d. over a point of support.

Answer: A

Assessment Questions

4. The center of gravity of an object
   a. lies inside the object.
   b. lies outside the object.
   c. may or may not lie inside the object.
   d. is near the center of mass.

Assessment Questions

4. The center of gravity of an object
   a. lies inside the object.
   b. lies outside the object.
   c. may or may not lie inside the object.
   d. is near the center of mass.

Answer: C

Assessment Questions

5. An unsupported object will topple over when its center of gravity
   a. lies outside the object.
   b. extends beyond the support base.
   c. is displaced from its center of mass.
   d. lowers at the point of tipping.

Assessment Questions

5. An unsupported object will topple over when its center of gravity
   - a. lies outside the object.
   - b. extends beyond the support base.
   - c. is displaced from its center of mass.
   - d. lowers at the point of tipping.

Answer: B

Assessment Questions

6. The center of gravity of your best friend is located
   a. near the belly button.
   b. at different places depending on body orientation.
   c. near the center of mass.
   d. at a fulcrum when rotation occurs.

Assessment Questions

6. The center of gravity of your best friend is located
   a. near the belly button.
   b. at different places depending on body orientation.
   c. near the center of mass.
   d. at a fulcrum when rotation occurs.

Answer: B

Assessment Questions

7. When a stable object is made to topple over, its center of gravity
   a. is at first raised.
   b. is lowered.
   c. plays a minor role.
   d. plays no role.

Assessment Questions

7. When a stable object is made to topple over, its center of gravity
   - a. is at first raised.
   - b. is lowered.
   - c. plays a minor role.
   - d. plays no role.

Answer: A